Model Question Paper
Mathematics
Class XII

Time Allowed : 3 hours | Maximum Marks: 100

General Instructions

(i) The question paper consists of three parts A, B and C. Each question of each part is compulsory.

(ii) **Part A**

Question number 1 to 20 are of 1 mark each.

(iii) **Part B**

Question number 21 to 31 are of 4 marks each

(iv) Question number 32 to 37 are of 6 marks each

Part A

Choose the correct answer in each of the questions from 1 to 20. Each of these question contain 4 options with just one correct option.

1. Let **N** be the set of natural numbers and R be the relation in **N** defined as $R = \{(a, b) : a = b - 2, b > 6\}$. Then

(A) $(2, 4) \in R$ (B) $(3, 8) \in R$

(C) $(6, 8) \in R$ (D) $(8, 7) \in R$.

2. If $\sin^{-1} x = y$, then

(A) $0 \leq y \leq \pi$ (B) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0 < y < \pi$ (D) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

3. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) 1

4. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

(A) 27 (B) 18

(C) 81 (D) 512

5. Let A be a square matrix of order 3×3, then $|kA|$ is equal to

(A) $k|A|$ (B) $k^2|A|$

(C) $k^3|A|$ (D) $3k|A|$

6. If $f(x)=\begin{cases}1+x, \text{ when } x \le 2\\ 5-x, \text{ when } x \le 3\end{cases}$, then

(A) f is continuous at $x = 2$ (B) f is discontinuous at $x = 2$

(C) f is continuous at $x = 3$ (D) f is continuous at $x = 3$ and discontinuous at $x=2$

7. $\frac{d}{dx}(\log \tan x)$ is equal to

(A) 2 sec $2x$ (B) 2 cosec $2x$

(C) sec $2x$ (D) cosec $2x$

8. The rate of change of the area of a circle with respect to its radius r when $r = 6$ is

(A) 10π (B) 12π

(C) 8π (D) 11π

9. The interval in which $y = x^2 e^{-x}$ is increasing is

(A) $(-\infty, \infty)$ (B) $(-2, 0)$

(C) $(2, \infty)$ (D) $(0, 2)$

10. $\int x \sec^2 x \, dx$ is equal to

(A) $\tan x^2 + C$ (B) $\tan^2 x + C$

(C) $x \tan x - \log \sin x$ (D) $x \tan x + \log \cos x + C$

11. $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ is equal to

(A) $\frac{-e^x}{x^2} + C$

(B) $\frac{-e^x}{x^2} + C$

(C) $\frac{-e^x}{x^2}$

(D) $\frac{e^x}{x}$

12. Area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$ is

(A) 2sq. units

(B) 4 sq. units

(C) 8sq. units

(D) 16sq. units

13. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(A) $e^x + e^{-y} = C$

(B) $e^x + e^y = C$

(C) $e^{-x} + e^y = C$

(D) $e^{-x} + e^{-y} = C$

14. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

(A) e^{-x}

(B) e^{-y}

(C) $\frac{1}{x}$

(D) x

15. The vector $2\hat{i} + \alpha\hat{j} + \hat{k}$ is perpendicular to the vector $2\hat{i} - \hat{j} - \hat{k}$ if

(A) $\alpha = 5$

(B) $\alpha = -5$

(C) $\alpha = -3$

(D) $\alpha = 3$

16. If $\vec{u} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{v} = 6\hat{i} - 3\hat{j} + 2\hat{k}$, then a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$ is

(A) $\hat{i} - 10\hat{j} - 18\hat{k}$

(B) $\frac{1}{\sqrt{17}}\left(\frac{1}{5}\hat{i} - 2\hat{j} - \frac{18}{5}\hat{k}\right)$

(C) $\frac{1}{\sqrt{473}}\left(7\hat{i} - 10\hat{j} - 18\hat{k}\right)$

(D) $\frac{1}{\sqrt{425}}\left(\hat{i} - 10\hat{j} - 18\hat{k}\right)$

17. If the given lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, then k is

(A) –10

(B) $\frac{10}{7}$

(C) $\frac{-10}{7}$

(D) $\frac{-7}{10}$

18. The angle between the planes $\vec{r}.(3\hat{i}-4\hat{j}+5\hat{k})=0$ and $\vec{r}.(2\hat{i}-\hat{j}-2\hat{k})$ is

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

19. The probability of obtaining an even prime numbr on each die, when a pair of dice is rolled is

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{1}{12}$

(D) $\frac{1}{36}$

20. If **R** is the set of real numbers and f : **R**→**R** be the function defined by $f(x)$ = $(3-x^3)^{\frac{\;}{3}}$, then $(fof)(x)$ is equal to

(A) $x^{\frac{\;}{3}}$

(B) x^3

(C) x

(D) $(3-x^3)$

Part B

21. Let **R** be the set of real numbers, f: **R**→**R** be the function defined by $f(x) = 4x + 3, x \in$ **R**. Show that f is invertible. Find the inverse of f.

22. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}} \le x \le 1$

23. If x, y, z are different numbers and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$

24. Is the function f defined by $f(x) = \begin{cases} x+5, \text{if } x \leq 1 \\ x-5, \text{if } x > 1 \end{cases}$ a continuous function? Justify your answer.

25. If $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$. Find $\frac{d^2y}{dx^2}$

26. Find $\int (\sin^{-1} x)^2 \, dx$

OR

Find $\int \frac{x^2+1}{x^2-5x+6} \, dx$

27. Evaluate $\int_{-2}^{2} |1-x^2| \, dx$

28. Find the solution of the differential equation $(x^2 + y^2)\, dx = 2xy\, dy$

OR

Find the equation of the curve passing through the origin and satisfying the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$

29. If the magnitude of the vectors, $\overline{a}$, $\overline{b}$, $\overline{c}$ are 3, 4, 5 respectively and $\overline{a}$ and $\overline{b} + \overline{c}$, $\overline{b}$ and $(\overline{c} + \overline{a})$, $\overline{c}$ and $(\overline{a} + \overline{b})$ are mutually perpendicular, then find the magnitude of $(\overline{a} + \overline{b} + \overline{c})$

30. Find the equation of the straight line passing through (1, 2, 3) and perpendicular to the plane $x + 2y - 5z + 9 = 0$

OR

Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$

31. A man and his wife appear for an interview for two posts. The probability of

husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. Find the probability that only one of them will be selected.

Part-C

32. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

OR

If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2x = -3$$

33. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$

OR

An open box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

34. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

OR

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

35. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q

contains 3 units of calcium, 20 units of iron, 4 units of cholesterol, and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

36. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by any other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train.

OR

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

37. Find the distance between the point P (6, 5, 9) and the plane determined by points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6).